Mathematical Studies

2015 Subject Outline

Stage 2

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Introduction

Purposes of the SACE

The South Australian Certificate of Education (SACE) is designed to enable students to:

* develop the capabilities to live, learn, work, and participate successfully in a changing world
* plan and engage in a range of challenging, achievable, and manageable learning experiences, taking into account their goals and abilities
* build their knowledge, skills, and understanding in a variety of contexts, for example, schools, workplaces, and training and community organisations
* gain credit for their learning achievements against performance standards.

Subject Description

Mathematical Studies may be undertaken as a 20-credit subject at Stage 2. Students who complete this subject with a C grade or better will meet the numeracy requirement of the SACE.

Mathematics is a diverse and growing field of human endeavour. Mathematics makes a unique contribution to the understanding and functioning of today’s complex society. By facilitating current and new technologies and institutional structures, mathematics plays a critical role.

Individuals require many aspects of mathematics in order to function adequately as members of society. The unprecedented changes that are taking place in the world will profoundly affect the future of today’s students. The effective use of technology and the processing of large amounts of quantitative data are becoming more important. Mathematics is increasingly relevant to the workplace and in everyday life. The study of mathematics provides students with the abilities and skills to thrive now and in the future.

Mathematics is much more than a collection of concepts and skills; it is a way of approaching new challenges by investigating, modelling, reasoning, visualising, and problem-solving, with the goal of communicating to others the relationships observed and problems solved.

Mathematics enables students to identify, describe, and investigate the patterns and challenges of everyday living. It helps students to analyse and understand the events that have occurred and to predict and prepare for events to come so they can more fully understand the world and be knowledgable participants in it.

Mathematics is a universal language that is communicated in all cultures. It is appreciated as much for its beauty as for its power. Mathematics can be seen in patterns in nature and art, in the proportions in architecture, in the form of poetry, and in the structure of music. Mathematics describes systematic, random, and chaotic behaviour; it is about relationships, exploration, intuition, and strategy.

Mathematical Studies allows students to explore, describe, and explain aspects of the world around them in a mathematical way. It focuses on the development of mathematical skills and techniques to facilitate this exploration. It places mathematics in relevant contexts and deals with relevant phenomena from the students’ common experiences as well as from scientific, professional, and social contexts.

The coherence of the subject comes from its focus on the use of mathematics to model practical situations, and on its usefulness in such situations. Modelling, which links the three mathematical areas to be studied, is made more practicable by the use of electronic technology.

Capabilities

The aim of the SACE is to develop well-rounded, capable young people who can make the most of their potential. The capabilities include the knowledge and skills essential for people to act in effective and successful ways.

The five capabilities that have been identified are:

* communication
* citizenship
* personal development
* work
* learning.

The capabilities enable students to make connections in their learning within and across subjects in a wide range of contexts.

Aspects of all the capabilities are reflected in the learning requirements, the content, the assessment design criteria, and the performance standards. Mathematical Studies empowers students to better understand and describe their world, and changes in it. As a result, students appreciate the role mathematics can play in effective decision-making. This subject also caters for students who want to continue to learn mathematics, and opens up a range of different career options by addressing aspects of the capabilities for work and learning. Although communication is an explicit feature of the assessment design criteria and the performance standards, the problems-based approach provides opportunities for students to develop aspects of citizenship and personal development.

Communication

In this subject students develop their capability for communication by, for example:

* communicating mathematical reasoning and ideas to a range of audiences, using appropriate language and representations, such as symbols, equations, tables, and graphs
* interpreting and using appropriate mathematical terminology, symbols, and conventions
* analysing information displayed in a variety of representations and translating information from one representation to another
* justifying the validity of the results obtained through technology or other means, using everyday language, when appropriate
* building confidence in interpreting, applying, and communicating mathematical skills in commonly encountered situations to enable full, critical participation in a wide range of activities.

Students have opportunities to read about, represent, view, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students are encouraged to use different forms of communication while learning mathematics.

Communication enables students to make connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Students develop the ability to explore, to make and test conjectures, to reason logically, and to use a variety of mathematical methods to solve problems.

Citizenship

In this subject students develop their capability for citizenship by, for example:

* understanding how mathematics helps individuals to operate successfully in an emerging global, knowledge-based economy
* gaining knowledge and understanding of the ways in which mathematics can be used to support an argument or point of view
* acquiring mathematical skills that will enable students to become leaders in various fields of endeavour in society
* understanding the contribution of mathematics and mathematicians to society now and in the future
* learning to critique the ways in which the mass media present particular points of view
* understanding the mathematics involved in technologies and making informed decisions about their use.

In a time of major change, nations, states, and their citizens have to operate successfully in an emerging global, knowledge-based economy. Major social, cultural, and environmental changes are occurring at the same time as changing commercial relationships, and the introduction of new information and communication technologies and the more recently developed sciences and technologies. Mathematics plays an important part in all of these.

In Mathematical Studies the main emphasis is on developing students’ knowledge, understanding, and skills so that they may use their mathematics with confidence as informed citizens capable of making sound decisions at work and in their personal environments.

Students are living in a rapidly changing world where decisions are based on quantitative understanding and reasoning. Therefore it is important that they value the necessity and relevance of mathematics to lifelong learning.

Mathematics allows people to deal with aspects of reality and provides the language to describe certain phenomena. Students should be able to discuss mathematical ideas in a clear, concise manner.

Mathematics is contextual and relies upon agreements among people who use it. All citizens should learn to appreciate this aspect of mathematics as a worldwide intellectual and cultural achievement. Understanding the history of mathematics in their culture and using mathematics successfully celebrates this achievement and allows further evolution of mathematics.

Personal Development

In this subject students develop their capability for personal development by, for example:

* acquiring the capacity for inventive thought and problem-solving, using mathematical techniques
* gaining an appreciation of the value of mathematics to the lifelong learner
* making decisions informed by mathematical reasoning
* arriving at a sense of self as a capable and confident user of mathematics by expressing and presenting ideas in a variety of ways.

Students should be able to use mathematics as a tool to solve problems they encounter in their personal lives. Every student should acquire a repertoire of problem-solving strategies and develop the confidence required to meet the challenges of a rapidly changing world.

Technology offers a wide and ever-changing variety of services to individuals and enterprises. It is important therefore that individuals have confidence in their mathematical abilities to understand the services offered and to make informed judgments about them.

Work

In this subject students develop their capability for work by, for example:

* reaching an understanding of mathematics in a range of relevant work contexts
* understanding the role of mathematics in contemporary technological society
* acquiring the mathematical knowledge and skills required for the particular pathway chosen by the student.

The mathematical skills required in the workplace are changing, with an increasing number of people involved in mathematics-related work. Such work involves increasingly sophisticated mathematical activities and ways of thinking. Although the use of information technology has changed the nature of the mathematical skills required, it has not reduced the need for mathematics.

It is important that students have the opportunity to gain an understanding of mathematics that will allow them to be designers of the future and leaders in various fields. They may be involved in product design, industrial design, production design, engineering design, or the design of new financial and commercial instruments.

The same considerations apply to the new sciences, and the new technologies they support. As systems for information-searching, data-handling, security, genetic design, molecular design, and smart systems in the home and at work become more sophisticated, users need to have a basic fluency in mathematics, and the designers of such technologies will need to have increasing understanding of mathematics.

Mathematics is a fundamental component of the success, effectiveness, and growth of business enterprises. Employees at various levels and in many types of employment are required to use their mathematical skills. Workers taking on greater responsibility for their own work areas use a wide range of mathematical skills. Some mathematical skills are used subconsciously because they are embedded in other tasks.

Students who want to enter fields such as architecture, economics, and biological, environmental, geological, and agricultural science should study Mathematical Studies. Students envisaging careers in other related fields may also benefit from studying this subject. If studied in conjunction with Specialist Mathematics, Mathematical Studies will provide students with pathways into courses such as mathematical sciences, engineering, physical sciences, and surveying.

Learning

In this subject students develop their capability for learning by, for example:

* acquiring problem-solving skills, thinking abstractly, making and testing conjectures, and explaining processes
* making discerning use of electronic technology
* applying knowledge and skills in a range of mathematical contexts
* interpreting results and drawing appropriate conclusions
* understanding how to make and test projections from mathematical models
* reflecting on the effectiveness of mathematical models, including the recognition of strengths and limitations
* using mathematics to solve practical problems and as a tool for learning beyond the mathematics classroom.

The unprecedented changes that are taking place in today’s world are likely to have a profound effect on the future of students. To meet the demands of the world in which they live, students need to adapt to changing conditions and to learn independently and collaboratively. They require the ability to use technology effectively and the skills for processing large amounts of quantitative information. They need an understanding of important mathematical ideas; skills of reasoning, problem-solving, and communication; and, most importantly, the ability and the incentive to continue learning on their own.

Making connections to the experiences of learners is an important process in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students are able to value mathematics as useful, relevant, and integrated, and to confidently apply their knowledge and skills to making decisions.

Students need to solve problems requiring them to use prior learning in new ways and contexts. Problem-solving builds students’ depth of conceptual understanding.

Learning through problem-solving helps students when they encounter new situations and respond to questions of the type ‘How could I . . .?’ or ‘What would happen if . . .?’ Students develop their own problem-solving strategies by being open to listening, discussing, conjecturing, and trying different strategies.

Mathematical reasoning helps students to think logically and to make sense of mathematics. Students are encouraged to develop confidence in their abilities to reason and justify their mathematical thinking.

Literacy in Mathematical Studies

It is important that students are able to express, interpret, and communicate information and ideas. Stage 2 Mathematical Studies gives students opportunities to grow in their ability to read, write, and talk about situations involving a range of mathematical ideas.

The ability to shift between verbal, graphical, numerical, and symbolic forms of representing a problem helps people to formulate, understand, and solve the problem, and communicate information. Students must have opportunities in mathematics to tackle problems requiring them to translate between different representations, within mathematics and between other areas.

Students learn to communicate findings in different ways, including orally and in writing, and to develop ways of illustrating the relationships they have observed or constructed.

Numeracy in Mathematical Studies

Students who complete Stage 2 Mathematical Studies with a C grade or better will meet the numeracy requirement of the SACE.

Being numerate is increasingly important in contemporary technological society. Students today require the ability to reason and communicate, to solve problems, and to understand and use mathematics. Developing these skills helps students to become numerate.

Students have opportunities to further develop their numeracy skills through the study of Stage 2 Mathematical Studies. The problems-based approach, integral to the development of the mathematical models and the associated key ideas in each topic, ensures the ongoing development of mathematical knowledge, skills, concepts, and technologies in a range of contexts.

Becoming numerate involves developing the ability to understand, analyse, critically respond to, and use mathematical knowledge, skills, concepts, and technologies in a range of contexts that can be applied to:

* using measurement in the physical world
* gathering, representing, interpreting, and analysing data
* using spatial sense and geometric reasoning
* investigating chance processes
* using number, number patterns, and relationships between numbers
* working with graphical and algebraic representations, and other mathematical models.

Aboriginal and Torres Strait Islander Knowledge, Cultures, and Perspectives

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of
high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

* providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
* recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
* drawing students’ attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
* promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

Learning Scope and Requirements

Learning Requirements

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through their learning.

In this subject, students are expected to:

1. understand fundamental mathematical concepts, demonstrate mathematical skills, and apply routine mathematical procedures

2. use mathematics as a tool to analyse data and other information elicited from the study of situations taken from social, scientific, economic, or historical contexts

3. think mathematically by posing questions/problems, making and testing conjectures, and looking for reasons that explain the results

4. make informed and critical use of electronic technology to provide numerical results and graphical representations

5. communicate mathematically and present mathematical information in a variety of ways

6. work both individually and cooperatively in planning, organising, and carrying out mathematical activities.

These learning requirements form the basis of the:

* learning scope
* evidence of learning that students provide
* assessment design criteria
* levels of achievement described in the performance standards.

Content

Stage 2 Mathematical Studies is a 20-credit subject that consists of the following three topics:

* Topic 1: Working with Statistics
* Topic 2: Working with Functions and Graphs Using Calculus
* Topic 3: Working with Linear Equations and Matrices.

Each topic consists of a number of subtopics. These are presented in this subject outline, in two columns, as a series of key questions and key ideas side-by-side with considerations for developing teaching and learning strategies.

A problems-based approach is integral to the development of the mathematical models and associated key ideas in each topic. Through key questions teachers can develop the key concepts and processes that relate to the mathematical models required to address the problems posed. The considerations for developing teaching and learning strategies present suitable problems and guidelines for sequencing the development of the ideas. They also give an indication of the depth of treatment and emphases required. This form of presentation is designed to help teachers convey concepts and processes to their students in relevant social contexts.

The key questions and key ideas cover the prescribed areas for teaching, learning, and assessment in this subject. Note that the considerations for developing teaching and learning strategies are provided as a guide only. The material for the external examination will be based on key questions and key ideas outlined in the three topics; the applications described in the considerations for developing teaching and learning strategies may provide useful contexts for examination questions.

Topics

The aim of this subject is to give students the tools to explore, describe, and explain aspects of the world around them in a mathematical way. The subject focuses on the mathematics needed for this exploration. This mathematics can empower students to describe their world, and changes in it. As a result, students appreciate the role that mathematics can play in effective decision-making.

The interrelationships of the topics are indicated and used in relevant contexts involving mathematical, physical, and social phenomena.

In Topic 1: Working with Statistics, students move from asking statistically sound questions towards a basic understanding of how, and why, statistical decisions are made. The topic provides students with opportunities and techniques to examine argument and conjecture from a ‘statistical’ point of view. This involves working with categorical and interval data, discovering and using the power of the central limit theorem, and understanding the importance of this theorem in statistical decision-making about means and proportions.

In Topic 2: Working with Functions and Graphs Using Calculus, students gain a conceptual grasp of introductory calculus, and the ability to use its techniques in applications. This is achieved by working with various kinds of mathematical models in different situations, which provide a context for the examination and analysis of the mathematical function behind the mathematical model.

In Topic 3: Working with Linear Equations and Matrices, students use a system of equations as a model to represent problem situations, solve such representations, and interpret their solution(s) in the context of the original model. Working with the systems of linear equations is another context for modelling with mathematics in practical situations.

Topic 1: Working with Statistics

Subtopic 1.1: Normal Distributions

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Why are normal distributions important?* The variation in many quantities occurs in an approximately normal manner
* Normal distributions may be used to make predictions and answer questions that relate to such quantities
 | Students are reminded of the sets of data in Subtopic 7.8 of Stage 1 Mathematics and the bell-shaped distributions that frequently resulted. |
| Why do so many observed sets of data appear normally distributed?* Quantities that arise as the sum of a large number of independent random variables can be modelled as normal distributions
 | When investigating one explanation for why normal distributions occur, students can experience the building of a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers. Potentially useful examples can be found in manufacturing processes (e.g. the length of a brick paver). |
| When can a set of data be called approximately normally distributed?* The following features will be approximated when the data set is represented by a histogram

symmetry about the sample meanbell shape* For large sets of data the standard deviation can be used as an approximate measure for probabilities within the distribution

What is the normal distribution?  | Students could experience a range of data sets that illustrate both normal and non-normal characteristics.A refinement of the spreadsheet mentioned above will allow students to see the features of normal distributions unfold. Students can relate to the ‘’ rule. |
| It is a mathematical model for a family of curves with the following properties:* symmetric
* bell-shaped
* determined by the mean and the standard deviation
* the axis of symmetry is at
* the spread is determined by
 | It may be useful for students to investigate the equation of the normal curve: |
| * the points of inflection are one standard deviation either side of the mean
* the area under the curve is 1.
 | The point of inflection could be investigated in the calculus section of the course. |
| How is the normal distribution used?* When the normal distribution is used to model data, the area under the curve over an interval can be interpreted as a

proportionprobability* It can be used to determine the value above or below which a certain proportion lies
 | Students could compare the histogram of the data (both normally and non-normally distributed examples) with the normal curve with  and  The aim is to show that not all data can be modelled with a normal distribution. |
| Standardisation* All normal distributions can be transformed to the standard normal distribution with  and  by using the formula

 | Students develop an understanding that for all normal curves the value of does not depend on  and  and is equal to Standardisation can be used for the comparison of two quantities measured on different scales.can be interpreted as the number of standard deviations by which  lies above or below the mean.Standardised normal probability statements provide a basis for the derivation of confidence intervals in Subtopic 1.4. |

Subtopic 1.2: The Central Limit Theorem

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| When a sample is chosen, any statistic calculated from that sample (notably the mean) can be considered as a variable. Repeated sampling will generate the distribution of the statisticFor example, if the chosen statistic is the sample mean  the distribution of will be generated | Students are reminded of the process of taking a random sample from a population demonstrated in Subtopics 7.6 and 7.8 of Stage 1 Mathematics.Students start by taking random samples generated by, for example, rolling dice, and progress to using graphics calculators and/or computers to simulate and streamline the process of sampling. Once students understand the ideas behind the simulation they can take many samples from a population, calculate their means, and analyse the distribution of the means. The sizes of samples taken are varied so that the characteristics of sampling distributions can be induced. |
| Suppose a random sample of size  is taken from a parent distribution of mean and standard deviation A practical consequence of the central limit theorem is that, for sufficiently large  the distribution of  will be approximately normal with mean and standard deviationThis result applies to a wide range of parent distributions. The required size of *n* depends on the shape of the parent distribution.Assessment questions will state that the value of  provided is sufficiently large, or students will be given data that allow them to demonstrate this fact | The use of electronic technology for repeated sampling from numerous populations of various forms (e.g. normal, skewed, U-shaped, and uniform) results in many sample means so that students can induce relevant aspects of the central limit theorem. This process also illustrates that assessing the required size of  is not a trivial task. The closer the parent distribution is to normal, the smaller the required value of  will be. When the parent distribution is normal, the distribution of  will be normal for all  |
| What is the importance of the central limit theorem for statistics?* Probabilities for  can be calculated using the normal distribution, whether or not the parent distribution is normal (given that  is sufficiently large)
 | Students could explore problems such as calculating the likelihood of buying an individual can of soft drink that contains less than 375 millilitres compared with the likelihood of buying a six-pack of the same soft drink with an average content of less than 375 millilitres. |

Subtopic 1.3: Testing Claims about a Population Mean 

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How can claims about the mean of a normal population with known standard deviation be tested from a simple random sample?In this subject two-tailedare used to assess claims about an unknown population meanGiven a simple random sample, the key concepts are:* null hypothesis
* alternative hypothesis
* null distribution
* test statistic
*
* level of significance (Fisher’s 0.05).

In the case of testing  for the population mean  the test statistic is  | It is critical that students review the differences between a sample and a population. Claims are often made that the mean of some quantity related to a population is a certain value. The truth of the claim can be known only by measuring the quantity for each member of the population, a method that is neither logistically nor economically feasible. For example, the mean volume of all cans of a given type of soft drink is claimed to be 376 millilitres; by taking one random sample from the population and observing the sample mean (e.g. 374 millilitres), it is possible to statistically retain or reject the claim.The probability of achieving what was observed or a more extreme result, calculated assuming the null hypothesis to be true, is called a ‘’.For example, if the chance of achieving 374 millilitres or a more extreme result is  then either a very rare event occurred or the original claim is untrue*.*of less than 0.05 are usually considered sufficient evidence for the rejection of the null hypothesis. Themeasures the weight of statistical evidence against a null hypothesis; the smaller the the more weight given to ‘evidence against’. |
| The conclusion of the hypothesis test is to determine whether or not there is sufficient evidence to reject the null hypothesis | A null hypothesis is rejected when there is sufficient evidence against it in the data.A null hypothesis is not rejected when there is insufficient evidence against it.When a null hypothesis is not rejected, the terms ‘retain’ and ‘accept’ are used in some statistical texts. This usage is technical and the more familiar meanings of these words do not apply. Outside this context, ‘accept’ does not represent a possible conclusion to a hypothesis test. |
| In this subject, when the sample size is sufficiently large, thecan be used more generally:* for non-normal populations
* when is not known and the sample standard deviation  is used as an estimate.

In these cases the test statistic and the will be approximate |  |
| If the null hypothesis is rejected, the direction of the difference will be determined by the value of the sample mean | Note that, if a null hypothesis is rejected, the application of a two-tailed test does not preclude the possibility of concluding the direction of the difference. |
| What information does the test not give?If the null hypothesis is rejected, the extent of the inaccuracy of the claim is still not known | This leads straight to the need for a ‘how much’ statement about a population mean (i.e. a confidence interval).A possible extension is to consider cases of such large sample size that any statistical difference found may not be of practical importance. |

Subtopic 1.4: Confidence Intervals for a Population Mean  for Interval Data

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How can the mean of a normal population with known standard deviation be estimated from a simple random sample?* The sample mean  provides an estimate of the population mean  that is subject to random error
* The distribution of  is normal
* The likely size of the error is determined by (the standard error)
 | By taking a random sample from a population and determining its sample mean, it is possible to determine an interval within which, at some level of confidence, the true population mean will lie. |
| The uncertainty in the estimate is described by specifying a range of plausible values for the population mean (a 95% confidence interval for ) | The algebraic derivation of the confidence interval comes directly from a standard normal probability statement about where the random variable  is the mean from a sample of observations drawn from a population with mean  and standard deviation  |
| These confidence intervals can be calculated directly using electronic technologyIn this subject, when the sample size is sufficiently large, the confidence interval can be used as an approximation more generally:* for non-normal populations
* when is not known and the sample standard deviation  is used as an estimate.

How can a 95% confidence interval forbe interpreted?* The practical interpretation is that there can be 95% confidence that the interval contains the population mean

A confidence interval can be used as a guide to the validity of a claim about a population mean | The notion of 95% confidence can be developed through repeated sampling from a given population with known  For each of a large number of samples, a confidence interval for can be calculated. It will be seen that approximately 95% of the intervals contain A graphical presentation of these ideas may be helpful. |
| Other simple applications are:* the effect of the sample size  on the width of the confidence interval
* calculation of the approximate sample size required to return a confidence interval of a desired width

 |  |

Subtopic 1.5: Binomial Distributions

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Why are binomial distributions important?* Many situations can be thought of as having only two outcomes (e.g. success/failure, yes/no)
* In many cases it is of interest to be able to predict the total number of successful outcomes
* Such predictions can be obtained using the binomial distribution
 |  |
| How are these situations modelled so that predictions can be made?* Using a tree diagram that leads to the formula

 | Students could start by using tree diagrams to explore success/failure situations. They see that tree diagrams become more cumbersome as the number of trials increases. The patterns evident in the trees could be seen as a link to a simpler representation.Counting combinations are then addressed in the context in which the students are working. It is not envisaged that these ideas would be derived from an in-depth exploration of ‘counting’. It is now a small step to the binomial formula  |
| What is the binomial distribution?It is the distribution of the total number of successes  from  independent trials, where each trial has the same probability of successWhen can a situation be modelled using the binomial distribution?* The number of trials  is fixed in advance
* The trials are independent
* Each trial has the same probability of success

How are probabilities calculated from binomial distributions?In this subject it is expected that electronic technology will be used | In a context where sampling is done without replacement (e.g. an opinion poll), the distribution of  is not strictly binomial. When the population is large in comparison with the sample, the binomial distribution provides an excellent approximation. |
| Are the binomial distribution and normal distribution related?* The normal distribution approximates the binomial distribution, provided that and
* The mean and standard deviation of a binomial count  are

 | Students could look at binomial distributions for increasingly large  and observe that even for values of  the shape of the distribution has approximately normal features.Historically, the normal distribution was used to calculate binomial probabilities. Technology has largely made this unnecessary.and  is a convention. The value 10 is seen as a conservative value that may vary from source to source. |

Subtopic 1.6: Testing Claims about a Population Proportion 

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What is a population proportion?* The proportion  of elements in a population that have a given characteristic

How can  be interpreted as a probability?* If one element of the population is chosen at random then the chance that it has the given characteristic is
* Given the correct conditions (see Subtopic 1.5), can be interpreted as the probability of success in a binomial trial

What is a sample proportion?If a sample of size  is chosen, and *X* is the number of elements with a given characteristic, then the sample proportion is equal to What is the distribution of ?In cases where  and  the distribution of  is approximately normalThe mean and standard deviation of a binomial proportion  are |  |
| and | These results can be arrived at by using the normal approximation to the binomial distribution, as seen in Subtopic 1.5. |
| How can claims about the population proportion be tested from a simple random sample?In this subject two-tailed  are used to assess claims about an unknown population proportion.The key concepts for a  are as given in Subtopic 1.3. | The approach required in this subtopic is essentially the same as that used in Subtopic 1.3. |
| In the case of testing  for the population proportion the test statistic is | In this test the  will be approximate. |
| If the null hypothesis is rejected, the direction of the difference will be determined by the value of the sample proportion. | Note that, if a null hypothesis is rejected, the application of a two-tailed test does not preclude the possibility of concluding the direction of the difference. |

Subtopic 1.7: Confidence Intervals for a Population Proportion 

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How can an unknown population proportionbe estimated from a simple random sample of size * The sample proportion  provides an estimate for  that is subject to random error
* The distribution of  is approximately normal provided that  and
* The likely size of the error is determined by  (the standard error)
* The uncertainty in the estimate is described by specifying a range of plausible values for the population proportion (a 95% confidence interval for)

These confidence intervals can be calculated directly using electronic technology | By taking a random sample from a population and determining its sample proportion, it is possible to determine an interval within which, at some level of confidence, the true population proportion will lie. The algebraic derivation comes directly from the normal approximation to the binomial distribution. |
| How can a 95% confidence interval for  be interpreted?* The interpretation is analogous to that for the mean (see Subtopic 1.4)
* There can be 95% confidence that the interval contains the population proportion
 | The notion of 95% confidence can be developed through repeated sampling from a given population with known  For each of a large number of samples, a confidence interval for can be calculated. It will be seen that approximately 95% of the intervals contain . |
| A confidence interval can be used as a guide to the validity of a claim about a population proportionOther simple applications are:* the effect of the sample size  on the width of the confidence interval
* calculation of the approximate sample size required to return a confidence interval of a desired width
 | A graphical presentation of these ideas may be helpful.  |
| where  is a given preliminary value for the proportion. | can be used to find  if no other value is available. This gives the most conservative value of  |

Topic 2: Working with Functions and Graphs Using Calculus

Subtopic 2.1: Using Functional Models

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Why are functional models used?* Models describe the change in one quantity in response to a second quantity, and can be used to predict or solve related problems

What types of models are to be considered and developed in the rest of this topic?* Polynomial of degree  (including linear)
* Rational of the form
* Power rational
* Exponential
* Logarithmic
* Surge
* Logistic
 | At this stage, students should be introduced to the range of models indicated here. There is a more formal treatment of functions in later subtopics. |
| How will models arise?* A known model is supplied
* A model can be derived by considering the structure (geometric or descriptive) of a problem
 | Students can be reminded of the work that they have done on modelling, and can re-examine models and their construction in contexts that require numerical, algebraic, and graphical approaches. This could be done in a number of ways:* numerical data graphical representation algebraic model

For example, by studying data from a real-world context and using electronic technology to graph the data points and develop an algebraic model for the data with the intention of using the model to interpolate and extrapolate* algebraic model numerical data graphical representation

For example, an algebraic model could be derived from assumptions or given properties and a table of values constructed using electronic technology. |
|  | Such a table could be used to:* observe a maximum, minimum, or zero value with a search-and-refine technique
* produce a graphical representation, which could be used for similar purposes.

It is envisaged that this approach would work effectively for:* quadratic models for the area of (say) rectangles with fixed perimeters
* rational models for the sides or perimeters of rectangles with fixed area.
 |
| What makes one model more appropriate than another?* Students need to be able to discuss the appropriateness of the model on the basis of its features and the structure of the problem or context

Maxima, minima, limiting behaviour (horizontal asymptotes), points of inflection, points of discontinuity (vertical asymptotes) | For example, modelling the spread of HIV and AIDS with an exponential function is clearly inappropriate over time. |

Subtopic 2.2: Computing Estimates of Areas Under Curves

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Why compute areas under curves?* To obtain useful information in modelled contexts
 | This subtopic can be introduced by getting students to revise the areas of polygons and then discussing the difficulty of generalising this idea to figures with curved boundaries. Students could start with simple graphs where area can be worked out using geometric principles (e.g. cross-sectional areas, and the determination of approximation of  or other irrational numbers). |
| When the rate of change of a quantity is graphed against the elapsed time, the area under the curve is the total change in the quantity | The following examples of rates of change models could be considered:* the rate at which people enter a sports venue
* water flow during a storm
* the velocity of a vehicle
* traffic flow through a city
* the electricity consumption of a household.
 |
| How can the area under a curve be estimated?* The area under a simple positive monotonic curve can be approximated by upper and lower sums (i.e. the sum of the areas of rectangles of equal width)

How does the exact area relate to the calculated underestimates and overestimates?* The exact area is between the two values

How can the estimate of the area be improved?* By decreasing the width of the rectangles
 | Problems introduced in Subtopic 2.1 may be approached using this approximation method before students learn how to formally integrate. Note that relatively few functions can be integrated, and numerical methods, similar to upper and lower sums, are used to estimate the area under a curve. |

Subtopic 2.3: Introduction to Exact Areas Under Curves

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Are there cases where the exact value of the area under a curve can be found?When the area is defined by a simple geometric shape (i.e. linear functions and sections of circles)* In cases where simple geometric shapes are not present, exact values can sometimes be inferred by the use of upper and lower sums or by using electronic technology
 | The area bounded by the graph of a positive function (not necessarily monotonic) and the  can be approximated by upper and lower sums. As the number of rectangles increases, the area is better approximated and the actual area always lies between these upper and lower sums. The exact area is defined as this unique number between all upper and lower sums. Examples of curves that could be considered include:* polynomial functions
*
*

The values of these areas can be estimated by using upper and lower sums* a semicircle or quarter-circle for which the exact area is ‘known’ (i.e. in terms of) and hence can be used to approximate
 |
| What is the exact value of the area under a curve?* The exact value of the area is the unique number between the upper and lower sums, which is approached as the width of the rectangles decreases
 | At this point it would be appropriate to pose the question ‘How could we prove that the exact values of area inferred in this subtopic are in fact correct?’* These areas can be determined exactly later, using integration techniques.
 |

Subtopic 2.4: Rate of Change

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What is a rate of change?* A rate of change is a ratio of the change in one quantity compared with that in a second, related, quantity
 | This idea can be covered in the context of, for example, finding average speeds, costs per kilogram, litres of water used per day, or watts of power used per day. To emphasise the connection between the quantities involved, the nature of averages could be explored by, for example, considering average rates of consumption of water, power, and so on, for one billing period, for different billing periods, for several billing periods, and for a year. |
| How can a constant rate of change be identified?* Numerically, in a table with a constant adder
* Algebraically, as a property of a linear function
* Graphically (and geometrically), as the gradient of a straight line
 | Useful contexts for exploring this important concept could be taken from examples such as: walking, running, cycling, or driving at a steady rate; filling a bucket from a mains-pressure tap; leaving a mains electrical appliance operating for a time; watering the garden.  |
| How can the rate of change of a non-linear function over an interval be considered?* The average rate of change of function  in the interval from  to  is

* The average rate of change of function in the interval from  to  is

* The average rate of change interpreted as the slope of a chord
 | Using some of the contexts already mentioned, the idea of non-constant rate of change can be explored by considering average rates over different time intervals for examples such as: an accelerating car; water delivered from a cask or dispenser (under gravity); and power delivered from a storage battery for a sufficiently long time to flatten the battery. As in the cases above, this concept can be strengthened by working:* numerically from tables of data
* algebraically from a formula
* graphically (and geometrically) by considering gradients of chords across graphs of curves (graphics calculators, interactive geometry, and graphing software provide invaluable visual support, immediacy, and relevance for this concept).

Applying all three approaches in one context would strengthen the presentation of this concept.To aid progression to future subtopics, students could explore how the average rate of change varies as the width of the interval decreases. |

Subtopic 2.5: Limiting Cases

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How do you approximate the rate of change at a point?* The rate of change across an interval is an approximation of the rate of change at a point (instantaneous rate of change)
* As the interval decreases, the approximation approaches the instantaneous rate of change (also to be interpreted as a chord approaching a tangent)
 |  |
| The instantaneous rate of change of a function at a point is the limit of the average rate of change over an interval that is approaching zero. | The notion of a limit can be motivated by considering graphs of the formor similar.Alternatively, the notion of a limit can be motivated by attempting to evaluate the fraction as  approaches zero. |

Subtopic 2.6: The Derivative

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| The derivative as an instantaneous rate of change | The derivative can be introduced as a summary of the ideas on rates of change and as a way to calculate an instantaneous rate of change. |
| Finding the derivative at a given point from first principles | and/or |
| Finding the derivative function from first principles | Use first principles to find the derivatives of functions such as at particular points and as functions.Use the first-principles approach to find the derivative of  for integer values of as an introduction to the development of the rules of differentiation. |
| Finding the derivative of  where  is a real constant |  |

Subtopic 2.7: Differentiation

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Algebraic structure of functions | Functions can be classified as sums, compositions, products, or quotients of simpler functions through an analysis of the use of grouping symbols (brackets) and a working knowledge of the order of operations. This is needed for the correct differentiation of the function. Students may need to revise some of the material covered in Subtopics 10.1 to 10.3 of Stage 1 Mathematics. |
| Differentiating sums of the form | The use of differentiation by first principles for a number of examples of simple polynomials helps in developing the rule |
| Differentiating composite functions of the form, with at most one application of the chain rule | The chain rule can be verified for a number of simple examples by establishing numerical results for the function’s derivative, using electronic technology, and establishing the same results algebraically by evaluating the product of the two factors proposed by the chain rule. An algebraic approach would be to apply the chain rule and simplify, then simplify and differentiate term by term. |
| Differentiating functions of the form, using the product rule | Methods similar to those used to introduce the chain rule could be used for the product rule. A more general justification could follow and allow progression to using the rule when the alternative expansions are too cumbersome or not available. |
| Differentiating functions of the form  using the quotient rule | The quotient rule can be established by applying the product rule to  |
| Implicit differentiation of relations | The slope of the tangent at a point on a circle can be established geometrically, as it is known that the tangent is perpendicular to the radius at the point of contact; hence, for a circle centred at the origin:  |
|  | With the formula established geometrically, the technique of implicit differentiation can be used to reproduce the result and provide a useful technique for relations that are less tractable geometrically. |

Subtopic 2.8: Using Derivatives

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Solving problems that use polynomial, rational, power models, simple composites of these, and implicit functions, involving the following concepts:* tangents and normals
* rates of change

increasing and decreasing functionsincreasing and decreasing rates of change (convexity/concavity) | Students would now be able to use functional models and their derivatives in the given contexts. |
| maxima and minima, local and globalstationary pointssign diagram of the first derivativeend points* points of inflection

turning point of the first derivativesign diagram of the second derivative. | In cases where non-exact values are required, the use of electronic technology should be encouraged. |
| The only application for which prior knowledge will be assumed is motion: displacement, velocity, and acceleration. | The use of displacement functions and their first and second derivatives: object changes direction when velocity changes sign; object is at rest when velocity is zero; object’s speed is increasing when velocity and acceleration have the same sign. |
| Other applications would include an explanation of the context within the question. | The following are examples of contexts that could be used:* economics
* population dynamics
* energy consumption
* water use
* drug concentration.
 |

Subtopic 2.9: Change and the Exponential Function

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What form does the derivative function of an exponential function  take?* It is also an exponential function that is a multiple of the original function

For what value of is  and  the same function? | Working in the context of a population with a simple doubling rule  and using electronic technology, it is possible to show that the derivative seems to be a multiple of the original growth function. This leads to a number of ‘What if …’ questions:* What if you investigated the derivative numerically by examining

for smaller and smaller values of * What if you considered the function
* What if you could find a base that provided for

Numerical and graphical investigations of the expression also lead to the existence of the number conventionally called  |

Subtopic 2.10: and  as Inverse Functions

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Given  what value of  will produce a given value of  | Examining a table of the non-negative integer powers of *e* gives another opportunity to witness the extent and rapidity of exponential growth, and also raises the following questions:* In a population growth model, when will the population reach a particular value?
* In a purely mathematical sense, what power of *e* gives 2, or 5, and so on? Looking for some of these values (by trial with a calculator) gives approximations for some specific natural logarithms, encountered previously as the multipliers for the derivatives of  and
 |
| The relationship between the graphs of  including their asymptotic behaviourand the relationships*
*
*
*

and their use in solving exponential equations. | These values can be encountered again when tracing a graph of the inverse of  and the natural logarithmic function can be introduced as the inverse of the exponential function.  |

Subtopic 2.11: Derivatives of Exponential and Logarithmic Functions

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| The derivative of:*
*
*
*
 | Studying the slope of tangents along the curve  leads to the hypothesisin a fairly straightforward fashion. Implicit differentiation can be used to confirm this hypothesis. |

Subtopic 2.12: Using Exponentials and Logarithms

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| Solving problems that use exponential and logarithmic models (and simple composites of these and polynomial, rational, and power models). Two specific examples are the surge and logistic models. These problems would involve the use of the following concepts:* tangents
* rates of change

increasing and decreasing functionsincreasing and decreasing rates of change (convexity/concavity)* maxima and minima, local and global

stationary pointssign diagram of the first derivativeend points* points of inflection

turning point of the first derivativesign diagram of the second derivative* limiting behaviour (horizontal asymptote).
 | The following examples of contexts could be used:* spread of disease (logistic model)
* flooding (surge model)
* drug concentration (surge model)

time to peak concentration (maximum)* market uptake of new products (logistic model)

point of diminishing returns (point of inflection)market saturation (horizontal asymptote). |

Subtopic 2.13: The Definite Integral

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| The exact value of the area under some curves can be determined. The exact area under other curves cannot. | The ideas involving the calculation of the exact value of the area under a curve from Subtopic 2.3 can be revised. |
| What is the definite integralfor a positive continuous function?* It can be interpreted as the exact area of the region between the curve  and the  over the interval
 | Students should be introduced to the correct notation and use electronic technology to find the approximate values for some integrals for which the exact values can be found later.This subtopic can be developed without reference to the algebraic computation of the antiderivative. |
| What is the definite integralfor a negative continuous function?* It can be interpreted as the negative of the exact area of the region between the curve  and the  over the interval
 |  |
| How can the definite integralbe interpreted for any continuous function ?* It can be interpreted as the sum of the definite integrals taken separately over the positive and negative parts of the function
 | The computation of carefully chosen integrals (using electronic technology) should lead students to observe the following results, which will be confirmed later:*
*
 |
| The definite integralcan alsobe interpreted as the net change in a quantity from  if  is the rate of change of the quantity with respect to  | Students should explore applications at this stage, such as:* the relationship between velocity, displacement, and distance travelled
* total electricity consumption being the integral of the rate of consumption.
 |
|  | Carefully chosen applications should lead students to observe the following result, which will be confirmed later:The relationship between the rate of change and the integral anticipates the fundamental theorem of calculus. |
| The area under a curve on an interval from *a* to *x* expressed as a function of *x* iswhen  is positive over this interval | Given a function  it is possible to start at some point *a* and find the cumulative areas under the function from *a* to *x* in regularincrements. This generates a table of values that can be modelled with the area function Functions such as  (constant) and  and other linear functions, allow exact values to be found. Electronic technology can then be used to help students to compute approximations for the area under polynomials. Fitting models to the data should enable students to predict the area function  without having to collect more data.For polynomials, finding  makes the relationship between and obvious. |
| How do you find the exact area of the region between a function and the *x*-axis over an interval ?* The sum of the individual areas above and below the  over the interval
 | It is a logical extension to find the exact area of the region between two functions over a given domain. |
| The statement of the fundamental theorem of calculus,where  is such that  | The fundamental theorem of calculus can now be derived, or it can be developed from the work above. |
| Direct results of the fundamental theorem of calculus are: *
*
*
 |  |

Subtopic 2.14: The Indefinite Integral

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What is the indefinite integral?* Any function  such that  is called an indefinite integral of
* If  is an indefinite integral of  then so is for any constant *c*
* It is written as

How is  obtained using algebraic means?* By implementing the process of antidifferentiation

What types of functions are to be considered for integration?* Power functions, including
* Exponential functions
* Polynomial functions
* Composite functions of the form  where is a power, exponential, or polynomial function, and  is a polynomial of, at most, degree 3
 | This section answers the question of how to find the definite integral algebraically for a select group of functions.Indefinite integrals can be used with the fundamental theorem of calculus to evaluate definite integrals. |
| The only application for which prior knowledge will be assumed is motion: displacement, velocity, and acceleration.Other applications will include an explanation of the context within the question. | The exact value of various definite integrals can now be computed using the fundamental theorem of calculus. These are calculated in the context of finding areas or calculating change in physical, biological, economic, or social phenomena. |

Topic 3: Working with Linear Equations and Matrices

Subtopic 3.1: Using Linear Equations

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How can some problems be expressed as systems of linear equations?* Identify the variables involved
* Identify relationships implied by the contextual information
* Construct the linear equations from these relationships
 | Examples of applications can be found in social, economic, biological, physical, and mathematical contexts, and could include:* allocating manufacturing resources
* balancing chemical equations
* blending wine
* designing balanced diets
* the unique fit of a parabola to three points
* Bézier curves (see Topic 3: Vectors and Geometry of Specialist Mathematics).

Students should test possible solutions to systems of linear equations, using examples with unique and non-unique solutions. |

Subtopic 3.2: Solutions to Systems of Equations

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| How can systems of linear equations be solved?* By algebraic methods for systems with two or three variables

represent the system in augmented matrix formperform row operations leading to row-echelon formsolve by back-substitution* Using electronic technology for systems with up to five variables
 | The solution of  systems of linear equations by substitution and elimination can be reviewed and extended to larger systems.Arrays of detached coefficients should be introduced as an equivalent representation of systems of equations. Reduction to row-echelon form by row operations on the augmented matrix is a systematic way of moving towards a solution.The idea of detaching the coefficients will be reinforced by the input format when using electronic technology.Solutions can be verified by substitution into the original equations or by using electronic technology. |
| A system of linear equations may have:* a unique solution
* infinitely many solutions; parameters can be used to describe an infinite set of solutions
* no solution.
 | Applications should illustrate systems that have non-unique or non-existent solutions. Examples could come from:* fitting polynomial curves through points (e.g. parabolas through two, three, or four points)
* flows in traffic and/or electrical networks
* balancing systems of levers
* the solution (when possible) of magic squares.

Note*:* The application to finding and describing the intersection of a collection of planes in three dimensions is included in Topic 3: Vectors and Geometry of Specialist Mathematics. |

Subtopic 3.3: Matrices

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What is a matrix?* Order of matrices
 | Matrices can provide a useful representation of information in a wide range of contexts. From a given context students could put numerical information in tabular form with the rows and/or columns labelled to identify what the elements represent. |
| How can matrices be used to represent real- world situations? | Applications of addition and scalar multiplication may include inventory and stock control, and networks. |
| What operations can be applied to matrices?* Addition and subtraction
* Scalar multiplication
* Matrix multiplication
 | Multiplication can be introduced via:* applications involving stock matrices and cost matrices
* applications involving transition matrices (e.g. population dynamics or the changing distribution of market shares between companies)
* encoding messages via a matrix, with letters represented by integers
* denoting linear systems as  and verifying solutions to systems solved in Subtopic 3.2.
 |
| Matrix addition is both commutative and associativeMatrix multiplication is associative but not commutativeThe identity matrix for matrix multiplication | The identity matrix should be introduced via its properties:  |

Subtopic 3.4: The Inverse of a Matrix

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| What is the inverse of a square matrix The matrix  is defined by the propertyNot all square matrices have an inverseThe inverse is not defined for non-square matricesHow is the inverse matrix calculated when it exists? |  |
| Using the formulafor a matrix * Using electronic technology for matrices larger than
 | The formula for the inverse of a  matrix can be derived using row operations. |
| How can inverse matrices be used?* To find the unique solution to matrix equations of the form

,if it exists | The matrix equation  can be compared with the number equation Students could use inverse matrices to solve systems of linear equations with unique solutions from the applications considered in Subtopic 3.2.A useful example is that of matrix codes: encrypting via a matrix and decrypting via its inverse. |
| Is there an indicator of when a square matrix will have an inverse?* The inverse of a matrix exists if, and only if, its determinant is non-zero

How are determinants calculated?* Using formulae for  and  cases
* Using electronic technology
 |  |
| For the system if  does not exist, then there are either:* no solutions

or* infinitely many solutions.

For the system if  does exist, then  |  |

Assessment Scope and Requirements

All Stage 2 subjects have a school assessment component and an external assessment component.

Teachers design a set of school assessments that enable students to demonstrate the knowledge, skills, and understanding they have developed to meet the learning requirements of the subject. These assessments provide students’ evidence of learning in the school assessment component.

Evidence of Learning

The following assessment types enable students to demonstrate their learning in
Stage 2 Mathematical Studies:

School Assessment (70%)

* Assessment Type 1: Skills and Applications Tasks (45%)
* Assessment Type 2: Folio (25%)

External Assessment (30%)

* Assessment Type 3: Examination (30%).

Students should provide evidence of their learning through nine to twelve assessments, including the external assessment component. Students undertake:

* at least six skills and applications tasks
* at least two investigations for the folio
* one examination.

Assessment Design Criteria

The assessment design criteria are based on the learning requirements and are used by:

* teachers to clarify for the student what he or she needs to learn
* teachers and assessors to design opportunities for the student to provide evidence of his or her learning at the highest possible level of achievement.

The assessment design criteria consist of specific features that:

* students should demonstrate in their learning
* teachers and assessors look for as evidence that students have met the learning requirements.

For this subject the assessment design criteria are:

* mathematical knowledge and skills and their application
* mathematical modelling and problem-solving
* communication of mathematical information.

The specific features of these criteria are listed below.

The set of assessments, as a whole, must give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Mathematical Knowledge and Skills and Their Application

The specific features are as follows:

MKSA1 Knowledge of content and understanding of mathematical concepts and relationships.

MKSA2 Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find solutions to routine and complex questions.

MKSA3 Application of knowledge and skills to answer questions in applied and theoretical contexts.

Mathematical Modelling and Problem-solving

The specific features are as follows:

MMP1 Application of mathematical models.

MMP2 Development of solutions to mathematical problems set in applied and theoretical contexts.

MMP3 Interpretation of the mathematical results in the context of the problem.

MMP4 Understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.

MMP5 Development and testing of conjectures, with some attempt at proof.

Communication of Mathematical Information

The specific features are as follows:

CMI1 Communication of mathematical ideas and reasoning to develop logical arguments.

CMI2 Use of appropriate mathematical notation, representations, and terminology.

School Assessment

Assessment Type 1: Skills and Applications Tasks (45%)

Students undertake at least six skills and applications tasks.

Students find solutions to mathematical questions/problems that may:

* be routine, analytical, and/or interpretative
* be posed in familiar and unfamiliar contexts
* require a discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require the student to demonstrate an understanding of relevant mathematical ideas, facts, and relationships.

Students select appropriate algorithms or techniques and relevant mathematical information to find solutions to routine, analytical, and/or interpretative questions/problems. Some of these questions/problems should be set in a personal, global, or historical context.

Students provide explanations and arguments, and use notation, terminology, and representation correctly throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some questions/problems.

Skills and applications tasks are undertaken under the direct supervision of a teacher.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

* mathematical knowledge and skills and their application
* mathematical modelling and problem-solving
* communication of mathematical information.

Assessment Type 2: Folio (25%)

A folio consists of at least two investigations.

Note: Teachers may need to provide support and clear directions with the first investigation. However, subsequent investigation(s) should be less directed and set within more open-ended contexts.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of the investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

An investigation may be initiated by a student, a group of students, or the teacher. In some instances teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation. In other situations teachers may provide broad guidelines allowing the student or group of students sufficient scope to develop themes or aspects of their own choice. Teachers should be prepared to give some direction about the appropriateness of each student’s choice and to guide and support students’ progress in an investigation.

Students are encouraged to demonstrate their use of problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, and results are important considerations.

Students interpret and justify results, summarise, and draw conclusions. Students are required to give appropriate explanations and arguments in a report. An investigation may require the use of electronic technology.

An investigation provides an opportunity for students to work cooperatively to achieve the learning requirements. When an investigation is undertaken by a group, each student must submit an individual report.

A completed investigation should include:

* an introduction that outlines the problem to be explored, including its significance, its features, and the context
* the method required to find a solution, in terms of the mathematical model or strategy to be used
* the appropriate application of the mathematical model or strategy, including
* the generation or collection of relevant data and/or information, with details of the process of collection
* mathematical calculations and results, and appropriate representations
* the analysis and interpretation of results
* reference to the limitations of the original problem
* a statement of the results and conclusions in the context of the original problem
* appendices and a bibliography, as appropriate.

The format of an investigation may be written, oral, or multimodal.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

* mathematical knowledge and skills and their application
* mathematical modelling and problem-solving
* communication of mathematical information.

External Assessment

Assessment Type 3: Examination (30%)

Students undertake a 3-hour external examination based on the key questions and key ideas outlined in the three topics and their subtopics. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide useful contexts for examination questions/problems.

The examination consists of a range of questions/problems, some focusing on knowledge and routine skills and applications, and others on analysis and interpretation. Some questions/problems may require students to interrelate their knowledge, skills, and understanding in more than one topic. The skills and understanding developed through the investigations are also assessed in the examination.

Students must have access to approved electronic technology during the external examination. However, students need to be discerning in their use of electronic technology to find solutions to questions/problems in examinations.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

* mathematical knowledge and skills and their application
* mathematical modelling and problem-solving
* communication of mathematical information.

Performance Standards

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding, on the basis of the evidence provided, how well a student has demonstrated his or her learning.

During the teaching and learning program the teacher gives students feedback on, and makes decisions about, the quality of their learning, with reference to the performance standards.

Students can also refer to the performance standards to identify the knowledge, skills, and understanding that they have demonstrated and those specific features that they still need to demonstrate to reach their highest possible level of achievement.

At the student’s completion of study of each school assessment type, the teacher makes a decision about the quality of the student’s learning by:

* referring to the performance standards
* assigning a grade between A and E for the assessment type.

A SACE Board school assessment grade calculator is available on the SACE website (www.sace.sa.edu.au) to combine the grades for the school assessment.

In the external assessment, assessors use the performance standards to make a decision about the quality of students’ learning, based on the evidence provided.

The student’s school assessment and external assessment are combined for a final result, which is reported as a grade between A and E.

Performance Standards for Stage 2 Mathematical Studies

| - | Mathematical Knowledge and Skills and Their Application | Mathematical Modelling and Problem-solving | Communication of Mathematical Information |
| --- | --- | --- | --- |
| A | Comprehensive knowledge of content and understanding of concepts and relationships.Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions. Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Development and effective application of mathematical models.Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts.Concise interpretation of the mathematical results in the context of the problem.In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.Development and testing of valid conjectures, with proof. | Highly effective communication of mathematical ideas and reasoning to develop logical arguments.Proficient and accurate use of appropriate notation, representations, and terminology. |
| B | Some depth of knowledge of content and understanding of concepts and relationships.Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions.Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Attempted development and appropriate application of mathematical models.Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts.Complete interpretation of the mathematical results in the context of the problem.Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.Development and testing of reasonable conjectures, with substantial attempt at proof. | Effective communication of mathematical ideas and reasoning to develop mostly logical arguments.Mostly accurate use of appropriate notation, representations, and terminology. |
| C | Generally competent knowledge of content and understanding of concepts and relationships.Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions.Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Appropriate application of mathematical models.Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts.Generally appropriate interpretation of the mathematical results in the context of the problem.Some understanding of the reasonableness and possible limitations of the interpreted results, and some recognition of assumptions made.Development and testing of reasonable conjectures, with some attempt at proof. | Appropriate communication of mathematical ideas and reasoning to develop some logical arguments.Use of generally appropriate notation, representations, and terminology, with some inaccuracies. |
| D | Basic knowledge of content and some understanding of concepts and relationships.Some use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to routine questions.Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts. | Application of a mathematical model, with partial effectiveness.Partly accurate and generally incomplete solutions to mathematical problems set in applied or theoretical contexts.Attempted interpretation of the mathematical results in the context of the problem.Some awareness of the reasonableness and possible limitations of the interpreted results.Attempted development or testing of a reasonable conjecture. | Some appropriate communication of mathematical ideas and reasoning.Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy. |
| E | Limited knowledge of content.Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions.Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts with limited effectiveness. | Attempted application of a basic mathematical model.Limited accuracy in solutions to one or more mathematical problems set in applied or theoretical contexts.Limited attempt at interpretation of the mathematical results in the context of the problem.Limited awareness of the reasonableness and possible limitations of the results.Limited attempt to develop or test a conjecture. | Attempted communication of emerging mathematical ideas and reasoning.Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy. |

Assessment Integrity

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.edu.au).

Support Materials

Subject-specific Advice

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

Advice on Ethical Study and Research

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).